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Reliability Based Analysis of Contact Problems

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RELIABILITY BASED ANALYSIS OF CONTACT PROBLEMS

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SUMMARY

The Batdorf model is modified to include the reduction in shear due to the effect of compressive stresses on the crack face. This new formulation was used to obtain the probability of failure of ceramic components under contact stress conditions. The combined effect of the surface and volume flaws are included in the analysis. Due to the nature of the fracture of brittle materials under compressive loading, the component is modeled as a series system in order to establish bounds on the probability of failure.

INTRODUCTION

The problem of failure of two bodies in contact goes back to the period when railroad wheels, gears, and bearings were first used. The classical deterministic approach to design against failure is well accepted and widely used for metal components. However, because of the attractive physical and mechanical properties of modern ceramics (lightweight, high-temperature strength, and wear resistance), ceramic components have in recent years been considered for structural applications. As in the case of other brittle materials, ceramics exhibit a large variation in fracture stress that must be taken into account in design. This variation in strength results from the presence of microscopic random imperfections or flaws. Ceramic components contain two types of flaws: volume flaws and surface flaws. Volume flaws arise from material processing while surface flaws arise from grinding and other surface finishing operations.

Most of the probabilistic approaches to brittle fracture are formulated for tensile failure. However, ceramic components are being used under compressive loading, as in bearing applications. In this paper, the probability of failure under compressive loading is formulated based on the Batdorf tensile model.

REVIEW OF THE BATDORF MODEL

The failure of brittle materials is attributed to the presence of microscopic flaws. The material fails when the strength of the weakest flaw is exceeded. The weakest link model was proposed as a statistical theory of fracture (ref. 10). These flaws were assumed to be cracks whose strength was dependent on their size and orientation (ref. 3).

Batdorf assumed that these cracks were uniformly distributed and randomly oriented as shown in figure 1. The materials will fail when the effective stress on a crack reaches some critical value σ_{cr} characteristic of that

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particular crack. To determine the effective stress, assumptions are made concerning the shape and the fracture criterion (ref. 4).

Two of the mixed-mode fracture criteria are the maximum tensile strength and the strain-energy release rate. For this analysis, the strain-energy release rate criterion was selected because of its greater degree of shear-sensitivity. The effective stress applied on a crack is given as

$$\sigma_e^2 = \sigma_n^2 + \tau^2 \tag{1}$$

for a Griffith crack, where σ_n is the macroscopic stress normal to the plane of the crack and τ is the shear parallel to it. A crack will fracture when the effective stress is greater than or equal to the critical stress.

COMPRESSIVE MODEL

When the stress normal to the crack plane is compressive, the shear and the friction due to the normal stresses act against one another (ref. 1)

$$\sigma_e = \tau_e = \tau + \mu\sigma_n \tag{2}$$

where τ_e is the effective shear on the crack and μ is the internal friction coefficient of the material.

To determine the probability of failure, the material is divided into elements. Within each element the stress state is assumed constant. Each element contains a number of cracks. The crack density N is material dependent and is defined as the number of cracks per unit volume having a critical stress greater than or equal to σ_{cr}. This function is determined by experiment using uniaxial data (ref. 6). The probability of failure for volume flaws is

$$P_{fV} = 1 - P_{SV} = 1 - \exp\left(-\int_V dv \int_0^{\sigma_{emax}} \frac{dN}{d\sigma_{cr}} \frac{\Omega}{4\pi}\, d\sigma_{cr}\right) \tag{3}$$

where Ω is a solid angle containing all of the orientations for which $\sigma_e \geq \sigma_{cr}$. Then the quantity $\Omega/4\pi$ is the probability that the crack will lie within the angle since all orientations are considered equally likely. An example of this angle is shown in figure 2. A similar approach is taken to find the probability of failure for surface cracks (ref. 5)

$$P_{fA} = 1 - P_{SA} = 1 - \exp\left(-\int_A dA \int_0^{\sigma_{emax}} \frac{dN_s}{d\sigma_{cr}} \frac{\omega}{\pi}\, d\sigma_{cr}\right) \tag{4}$$

where N_S is the surface crack density function, A is the surface area, and ω is the radian measure of the angular range in the positive σ_1 half-plane within which $\sigma_e \geq \sigma_{cr}$. The total probability of survival, P_S, is equal to the probability of survival for the volume times the surface survival, $(P_{SV}P_{SA})$.

The upper limit for the integration of σ_{cr} is the maximum effective stress σ_{emax}. When $\sigma_{cr} > \sigma_{emax}$, Ω or ω is equal to zero. For tensile stress states the maximum effective stress is equal to the maximum principal stress. For compressive stress states, the maximum effective stress is dependent on the stress state and the internal friction coefficient. If $\sigma_e < 0$, the frictional force along the crack is greater than the shear and the crack is locked. This phenomenon is taking place near the orientation of minimum principal stress as illustrated in figure 2.

SYSTEM RELIABILITY UNDER COMPRESSIVE LOADING

Brittle materials with pre-existing cracks may fracture when loaded in triaxial compression (ref. 2). The unequal principal compressive stresses generate shear stresses which act against frictional forces, initiating local crack growth. Since the existing cracks are microscopic, a single crack does not produce total failure as it almost always does in tension. Total fracture occurs in compression when several of these cracks extend and join together (single crack arrest is a possibility under high compressive loading). To account for the multicracking phenomenon, the material is modeled as a series system rather than independent elements as in the weakest link theory. Limits are established on the probability of failure of the series system using the Ditlevsen bounds (ref. 7)

$$P_1 + \sum_{i=2}^{k} \max \left[P_i - \sum_{j=1}^{i-1} P_{ij}, 0 \right] \leq P_f \leq \sum_{i=1}^{k} P_i + \sum_{i=2}^{k} \max_{j<i} P_{ij} \tag{5}$$

where P_i is the probability of failure of an individual element, assembled in decreasing order, k is the number of elements and P_{ij} is the joint probability of failure of the elements i and j, which is formulated for simplicity in function of the safety indices β_i and β_j, given by

$$P_{ij} = P_i P_j + \int_0^{\rho} \varphi(-\beta_i, -\beta_j : z)\, dz \tag{6}$$

where

$$\varphi(x,y:\rho) = \frac{1}{\left(2\pi\sqrt{1 - p^2}\right)} \exp\left[-\frac{1}{2} \frac{(x^2 + y^2 - 2xy\rho)}{(1 - \rho^2)} \right] \tag{7}$$

The safety index $\beta_i = -\phi^{-1}(P_i)$ is a normally distributed function and ρ is the correlation coefficient ($\rho = 0$ no correlation and $\rho = 1$ fully correlated). When ρ is equal to one, a k series system is modeled as one single element whose probability of failure is the average of the k elements.

APPLICATION

The volume crack density function, N in equation (3), is assumed to be (ref. 3)

$$N = k_B \sigma_{cr}^m$$

where k_B and m are material constants. A similar expression, $N_s = k_{Bs}\sigma_{cr}^m$, was used as the surface crack density function in equation (4). These crack density functions were calculated based on compressive experimental data of alumina (ref. 9). To determine the crack density function the probability of failure versus the absolute value of the compressive strength was plotted. The resulting curve may be expressed (ref. 10)

$$P_f = 1 - \exp - \left[C|\sigma|^m\right] \tag{8}$$

where σ is the compressive strength and C and m are constants determined by linear regression. The constants k_B or k_{Bs} are determined by equating the exponent in equation (8) to the one in equation (3) or (4), respectively, for example

$$k_B = \frac{C|\sigma|^m}{\int_V \int_0^{\sigma_{emax}} m\sigma_{cr}^{m-1} \frac{\Omega}{4\pi} \, d\sigma_{cr} \, dv}$$

where $m = 43.41$, $k_B = 1.482 \times 10^5$ GPa $(m)^{0.069}$ and $k_{Bs} = 3.177$ GPa $(m)^{0.046}$.

The preceding theory was used to evaluate the probability of failure for an alumina ceramic under contact stress loading as shown in figure 3. A closed form solution to determine the stress distribution beneath the contact region was used (ref. 8). The probability of failure as a function of the maximum contact pressure is shown in figure 4 with $\mu = 0.35$.

When the elements are not correlated the weakest link probability lies within the Ditlevsen bounds. However, when the system is fully correlated, the probability of failure is substantially lower than the weakest link model, for a given load. This indicates that for the whole system to fail under compressive loading, more than one element needs to fail. An intermediate correlation function was assumed $\rho_{ij} = \exp -(r_{ij}/d - 1)$ where r_{ij} is the distance between the centroids of two elements i and j and d is the mesh size. This correlation is more realistic because ρ is equal to one for two adjacent elements and decreases as the distance between elements increases.

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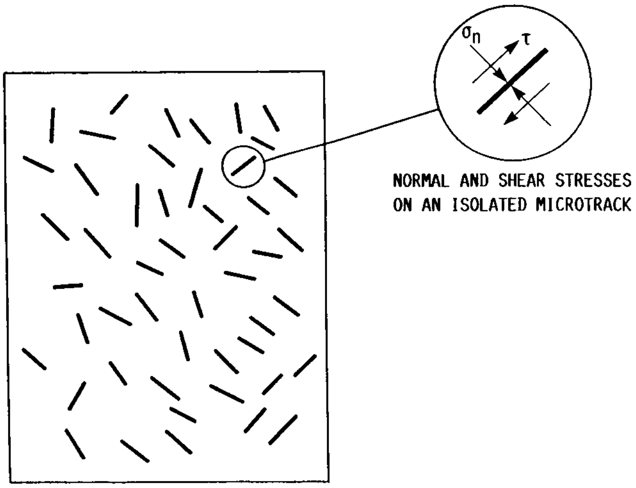

FIGURE 1. - RANDOM CRACK DISTRIBUTION IN BRITTLE MATERIALS.

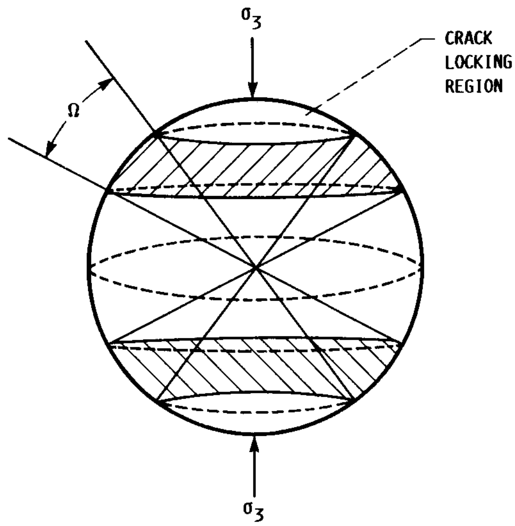

FIGURE 2. - SOLID ANGLE WITHIN WHICH THE NORMAL TO THE CRACK MUST BE ORIENTED FOR FRACTURE TO OCCUR. ($\sigma_1 = \sigma_2 = 0$, $\sigma_3 < 0$.)

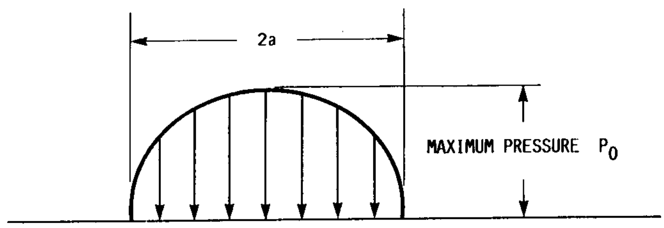

FIGURE 3. - SCHEMATIC VIEW OF A CONTACT STRESS DISTRIBUTION ON A SEMI-INFINITE REGION.

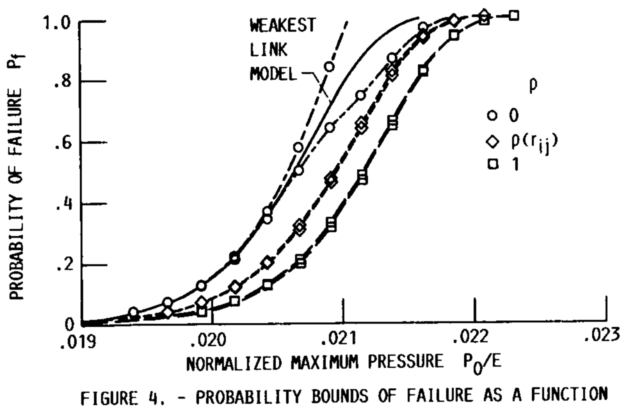

FIGURE 4. - PROBABILITY BOUNDS OF FAILURE AS A FUNCTION OF THE NORMALIZED MAXIMUM PRESSURE FOR DIFFERENT CORRELATION COEFFICIENTS, (ρ).

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16. Abstract

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